

# (3+2) Neutrino Scheme From A Singular Double See-Saw Mechanism

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We obtain a  $3 + 2$  neutrino spectrum within a left-right symmetric framework by invoking a singular double see-saw mechanism. Higgs doublets are employed to break  $SU_R(2)$  and three additional fermions, singlets under the left-right symmetric gauge group, are included. The introduction of a singularity into the singlet fermion Majorana mass matrix results in a light neutrino sector of three neutrinos containing predominantly  $\nu_{\alpha L}$ ,  $\alpha = e, \mu, \tau$ , separated from two neutrinos containing a small  $\nu_{\alpha L}$  component. The resulting active-sterile mixing in the  $5 \times 5$  mixing matrix is specified once the mass eigenvalues and the  $3 \times 3$  submatrix corresponding to the PMNS mixing matrix are known.

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## I. INTRODUCTION

Our understanding of neutrino masses and mixings has rapidly improved in recent years, with solar [1], atmospheric [2] and terrestrial [3, 4, 5, 6] neutrino oscillation experiments providing valuable insight. The reactor experiments CHOOZ [5] and Palo Verde [6] indicate that the atmospheric and solar oscillations are effectively decoupled [7] and the totality of the data suggests that the atmospheric and solar anomalies can be adequately explained by three flavour neutrino mixing. The reported  $\bar{\nu}_\mu - \bar{\nu}_e$  oscillation signal of LSND [8] provides an interesting piece of oscillation data that conflicts with this three flavour explanation. The ongoing MiniBooNE [9] experiment will soon test the LSND result.

The  $3 + 1$  and  $2 + 2$  neutrino spectra arose from a minimalist approach to the simultaneous resolution of the solar, atmospheric and LSND neutrino data in terms of neutrino oscillations. The neutrino spectrum is extended in a minimal fashion via the addition of one sterile neutrino state. Currently favoured fits to the solar and atmospheric data in terms of purely active neutrino oscillations leave little room for additional sterile states [10]. Recent high precision measurements of the S-factor (defined in [11]) by the Seattle group [12] give  $S_{17}(0) = 22.1 \pm 0.6$  eV-b, leading to an expected  ${}^8B$  solar neutrino flux 13% larger than that measured by SNO [13] (for a discussion see [14, 15]). Questions regarding the distinction between atmospheric  $\nu_\mu \rightarrow \nu_\tau$  and  $\nu_\mu \rightarrow \nu_s$  transitions [16] and the absolute statistical significance of some data fits [17] have also been raised. The  $3 + 1$  and  $2 + 2$  schemes come into conflict with the data in different ways. The source of incompatibility for  $2 + 2$  spectra comes from relations amongst the sterile components in the atmospheric and solar neutrinos that are difficult to reconcile with experimental results (it has been suggested that global fits to data that include the effects of small mixing angles, usually neglected in analysis, are re-

quired to invalidate the  $2 + 2$  schemes [18]).  $3 + 1$  spectra, on the other hand, are disfavoured by comparisons of short-baseline disappearance data [19, 20] with the LSND result.

The study of  $3 + 2$  spectra follows the minimalist attitude that motivated the four neutrino models and data fits. The addition of the second sterile state can simultaneously enhance the predicted LSND signal and relax the laboratory and atmospheric bounds on the mixing matrix elements  $U_{e4}$  and  $U_{\mu 4}$  [21]. The second sterile state is required to mix with both  $\nu_e$  and  $\nu_\mu$  to contribute to the LSND probability and avoid opening up new channels for  $\nu_e$  or  $\nu_\mu$  disappearance. Provided  $m_5^2 > \Delta m_{LSND}^2$  the bounds on  $U_{e4}$  and  $U_{\mu 4}$  are modified and the second  $\Delta m^2$  will contribute to LSND. The splitting between the two predominantly sterile states  $\Delta m_{45}^2$  should also be resolved by LSND to ensure the LSND signal is enhanced. If, for example,  $\Delta m_{14}^2 \sim (1 - 2)$  eV<sup>2</sup> and  $\Delta m_{15}^2 > 8$  eV<sup>2</sup>, the LSND signal can be enhanced whilst relaxing the short-baseline constraints [21]. The statistical analysis of [22] suggests that if  $\Delta m_{15}^2 \sim 22$  eV<sup>2</sup> the predicted LSND signal may be enhanced by 60-70%. Using horizontal symmetries,  $3 + 2$  spectra with see-saw suppressed light sterile neutrinos have been studied [23], whilst the coexistence of large active-active and large active-sterile mixing in  $3 + 2$  scenarios was studied in [24].

Though minimalist, the introduction of two sterile states seems counterintuitive to the suggestive demands of a familial quark-lepton symmetry. The latter makes the addition of three right-handed neutrinos to the standard model seem a logical extension. The discovery of a quark-lepton familial symmetry may hint at an underlying left-right symmetric gauge theory, broken to the standard model at some high energy scale. In this note our objective is to theoretically motivate a  $3 + 2$  neutrino model within a left-right symmetric framework. Higgs doublets are employed, rather than triplets, to break  $SU_R(2)$  and additional singlet neutral fermions, sterile under the gauge symmetries, are included. The  $3 + 2$  spectrum results from the introduction of a singularity into the singlet fermion Majorana mass matrix. The resulting modified double see-saw mechanism produces a light neutrino sector with three predominantly  $SU_L(2)$  active neutrinos separated from two neutrinos predominantly sterile under  $SU_L(2)$ .

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The structure of this note is as follows. In Section II the particle content of the model is presented in conjunction with a brief discussion of the double see-saw mechanism. The neutrino content of the model receives focus in Section III where the eigenstates are derived. Section IV contains a discussion of the scales required to make the resulting neutrino spectrum experimentally feasible and some concluding remarks can be found in Section V.

## II. EXTENDING THE STANDARD MODEL

The left-right symmetric model, with gauge group  $\mathcal{G}_{LR} = SU_C(3) \times SU_L(2) \times SU_R(2) \times U_{B-L}(1)$ , is considered a natural extension of the standard model (SM). The addition of three right-handed neutrinos to the SM fermionic spectrum automatically qualifies  $SU_R(2)$  as a gaugeable symmetry. The use of Higgs triplets to break  $SU_R(2)$  at a high energy scale provides a theoretical framework for the realisation of the see-saw mechanism [25], coupling the existence of light predominantly  $SU_L(2)$  doublet neutrinos to the existence of (as yet undetected) heavy gauge bosons [26]. An alternative path to massive neutrinos within a left-right symmetric framework relies on Higgs doublets, rather than triplets, to achieve the high energy breaking of  $SU_R(2)$  [27, 28]. This path denies the see-saw mechanism the opportunity to explain the lightness of the  $SU_L(2)$  active neutrinos, but the inclusion of extra neutral fermions, singlets under  $\mathcal{G}_{LR}$ , can provide an alternative explanation. The addition of three such singlets leads to a leptonic Yukawa lagrangian of the form:

$$\mathcal{L}_Y = h_{ij}^1 \bar{L}_L^i \phi L_R^j + h_{ij}^2 \bar{L}_L^i \tilde{\phi} L_R^j + M_{Sij} \bar{S}^{i^c} S^j + f_{ij} (\bar{L}_L^{ci} X_L S^j + \bar{L}_R^{ci} X_R S^j) + h.c., \quad (1)$$

where  $L_{L,R}$  are the fermion doublets,  $S$  denotes the singlet fermions,  $\phi$  is a Higgs bidoublet and  $X_{L,R}$  are Higgs doublets, ie:

$$L_L \sim (1, 2, 1, -1), L_R \sim (1, 1, 2, -1),$$

$$S \sim (1, 1, 1, 0),$$

$$\phi \sim (1, 2, 2, 0),$$

$$X_L \sim (1, 2, 1, 1), X_R \sim (1, 1, 2, 1).$$

Denoting the second Pauli matrix by  $\tau_2$  the bidoublet  $\tilde{\phi}$  in (1) is given by  $\tilde{\phi} = \tau_2 \phi^* \tau_2$ . The singlet neutrinos have bare Majorana mass terms, whilst the doublet neutrinos acquire Dirac mass couplings to the singlets only if  $X_{L,R}$  develop non-zero VEV's. One requires a non-zero value for  $\langle X_L \rangle$  to break  $SU_R(2)$  at some high scale, but may take  $\langle X_L \rangle = 0$  [28] to preclude Dirac mass terms coupling  $\nu_L^i$  to the singlets. In the basis  $(\nu_L, \nu_R^c, S^c)$  the neutral fermion mass matrix has the form:

$$\begin{pmatrix} 0 & m_{LR} & 0 \\ m_{LR}^T & 0 & M_{RS} \\ 0 & M_{RS}^T & M_S \end{pmatrix}, \quad (2)$$

where  $m_{LR}$  and  $M_{RS}$  are Dirac mass matrices and  $M_S$  is the singlet Majorana mass matrix. We shall denote the scale of non-zero entries in  $m_{LR}$ ,  $M_{RS}$  and  $M_S$  as  $m$ ,  $M$  and  $\mu$  respectively. The Dirac mass matrix  $m_{LR}$  ( $M_{RS}$ ) arises when  $\phi$  ( $X_R$ ) acquires a VEV. The physical condition  $\langle X_R \rangle \gg \langle \phi \rangle$  implies  $M \gg m$ , though the relationship between  $M$  and  $\mu$  is not predetermined. The effective light neutrino mass matrix is given by  $M_\nu = -m_{LR}^T (M_{RS}^{-1})^T M_S (M_{RS}^{-1}) m_{LR}$ . Some interesting scale hierarchies are:

- $\mu \ll M$

This provides a further suppressing factor of  $\frac{\mu}{M}$  to the light neutrino mass scale relative to that of the see-saw mechanism,  $\sim \frac{m^2}{M}$ . Consequently the suppressing scale  $M$  can be set lower. This case has been referred to as an inverse see-saw mechanism (see for example [29]). The small value required of  $\mu$  in this case is considered natural in the technical sense [30] as lepton number conservation is restored in the limit  $\mu \rightarrow 0$ .

- $\mu \gg M$

This hierarchy generates a see-saw mechanism between the  $\nu_R$ 's and the  $S$ 's, giving an effective right-handed neutrino scale of order  $\frac{M^2}{\mu}$ .

We note that neutrino mass matrices of this form are found in some string inspired models [31] and provide a basis for the so called double see-saw mechanism [32, 33].

## III. SINGULAR DOUBLE SEE-SAW MECHANISM

In this paper we investigate the double see-saw mechanism, with the hierarchy  $\mu \gg M$ , when the singlet Majorana mass matrix  $M_S$  is of rank 2. The Dirac mass matrices  $m$  and  $M$  will remain general. The  $9 \times 9$  mass matrix (2) will then lead to 9 Majorana neutrinos as follows:

- Two ultra-heavy Majorana neutrinos of order  $\mu$ , predominantly containing the fully sterile singlets,
- A pseudo-Dirac pair of heavy neutrinos of order  $M$ . These will be an admixture of the right handed neutrinos and the massless singlet,
- A lighter pair of Majorana neutrinos with mass  $\sim M^2/\mu$ , containing mostly  $\nu_R$ 's with a small  $\nu_L$  component,
- Three Majorana neutrinos of order  $m^2/(M^2/\mu)$ . These will be mostly  $\nu_R$ 's with a small  $\nu_L$  component.

We begin by performing a singular see-saw analysis [34, 35, 36] on the submatrix:

$$\begin{pmatrix} 0 & M_{RS} \\ M_{RS}^T & M_S \end{pmatrix}. \quad (3)$$

The  $9 \times 9$  mass matrix is repartitioned and the submatrix  $M_S$

diagonalised as follows:

$$M_{\nu(9 \times 9)} = \begin{pmatrix} 0 & m_{LR} & 0 \\ m_{LR}^T & 0 & M_{RS} \\ 0 & M_{RS}^T & M_S \end{pmatrix} \equiv \begin{pmatrix} A_{6 \times 6} & \beta_{6 \times 3} \\ (\beta_{6 \times 3})^T & M_S \end{pmatrix}$$

$$= \begin{pmatrix} I_{6 \times 6} & 0_{6 \times 3} \\ 0_{3 \times 6} & R_1^T \end{pmatrix} \begin{pmatrix} A & \beta R_1^T \\ R \beta^T & M_S^{diag} \end{pmatrix} \begin{pmatrix} I_{6 \times 6} & 0_{6 \times 3} \\ 0_{3 \times 6} & R_1 \end{pmatrix},$$

where  $M_S^{diag}$  has the zero eigenvalue of  $M_S$  in the (1,1) entry. A further repartition to separate the zero eigenvalue of  $M_S$  gives:

$$M_{\nu} = \begin{pmatrix} A & \beta R_1^T \\ R \beta^T & M_S^{diag} \end{pmatrix} = \begin{pmatrix} A'_{7 \times 7} & B_{7 \times 2} \\ (B_{7 \times 2})^T & \omega_{2 \times 2} \end{pmatrix},$$

where  $A'$  has the zero eigenvalue of  $M_S$  in its lower right corner. Next, we block diagonalise  $M_{\nu}$ :

$$\begin{aligned} M_{\nu} &= \begin{pmatrix} A'_{7 \times 7} & B_{7 \times 2} \\ B_{2 \times 7}^T & \omega_{2 \times 2} \end{pmatrix} \\ &= S \begin{pmatrix} Q_{7 \times 7} & 0_{7 \times 2} \\ 0_{2 \times 7} & \omega_{2 \times 2} \end{pmatrix} S^T, \end{aligned} \quad (4)$$

where:

$$S = \begin{pmatrix} I_{7 \times 7} & P_{1(7 \times 2)} \\ -(P_{1(7 \times 2)})^T & I_{2 \times 2} \end{pmatrix},$$

and:

$$P_1 = B \omega^{-1},$$

$$Q = A' - P_1 \omega P_1^T = A' - B \omega^{-1} B^T. \quad (5)$$

Equation (4) demonstrates that to order  $M^2/\mu$  the eigenvalues of  $M_{\nu}$  include two heavy Majorana neutrinos, with masses of order  $\mu$ , which are linear combinations of the sterile states  $S^i$ . The second term in the above expression for  $Q$  provides a seesaw type correction to the submatrix  $A'$ . The matrix  $Q$  has the form:

$$Q = \begin{pmatrix} 0_{3 \times 3} & \gamma_{3 \times 4} \\ (\gamma_{3 \times 4})^T & \omega_{2(4 \times 4)} \end{pmatrix},$$

where the non-zero elements of  $\gamma$  are of order  $m$  and  $\omega_2$  contains non-zero elements of order  $M$  and order  $M^2/\mu \ll M$ . The matrix  $\omega_2$  must now be diagonalised. A perturbative treatment gives the zeroth order eigenvalues as  $\omega_2^{diag} = (0, 0, \lambda_{\omega_2}^{(0)}, -\lambda_{\omega_2}^{(0)})$ , where  $\lambda_{\omega_2}^{(0)} \sim M$ . The eigenvectors corresponding to the zero eigenvalues are linear combinations of the  $\nu_R$ 's, whilst the Dirac pair contains the orthogonal combinations of  $\nu_R$ 's and the zero eigenvector of  $M_S$ . The correction term  $-B \omega^{-1} B^T$  of equation (5) splits the degenerate non-zero eigenvalues, forming a pseudo-Dirac pair, and gives a mass of order  $M^2/\mu$  to the zero eigenvalues. The diagonalisation of  $\omega_2$  gives:

$$Q = \begin{pmatrix} 0_{3 \times 3} & \gamma_{3 \times 4} \\ \gamma_{4 \times 3}^T & \omega_{2(4 \times 4)} \end{pmatrix}$$

$$= \begin{pmatrix} I_{3 \times 3} & 0_{3 \times 4} \\ 0_{4 \times 3} & R_{2(4 \times 4)}^T \end{pmatrix} \begin{pmatrix} 0_{3 \times 3} & \gamma R_2^T \\ R_2 \gamma^T & \omega_2^{diag} \end{pmatrix} \begin{pmatrix} I_{3 \times 3} & 0_{3 \times 4} \\ 0_{4 \times 3} & R_{2(4 \times 4)} \end{pmatrix},$$

and we repartition the above matrix to:

$$\begin{pmatrix} 0_{3 \times 3} & \gamma R_2^T \\ R \gamma^T & \omega_2^{diag} \end{pmatrix} = \begin{pmatrix} \Omega_{5 \times 5} & B_2 \\ B_2^T & \omega'_2 \end{pmatrix}, \quad (6)$$

where  $\omega'_2$  contains the two eigenvalues of order  $M$  from  $\omega_2^{diag}$  and the order  $M^2/\mu$  eigenvalues form the lower right block of  $\Omega$ . Finally we block diagonalise (6):

$$\begin{pmatrix} \Omega_{5 \times 5} & B_2 \\ B_2^T & \omega'_2 \end{pmatrix} = \mathcal{A} \begin{pmatrix} \Omega'_{5 \times 5} & 0 \\ 0 & \omega'_2 \end{pmatrix} \mathcal{A}^T \quad (7)$$

where:

$$\mathcal{A} = \begin{pmatrix} I_{5 \times 5} & P_{2(5 \times 2)} \\ -P_{2(2 \times 5)}^T & I_{2 \times 2} \end{pmatrix}, \quad (8)$$

and:

$$P_2 = B_2 \omega'^{-1},$$

$$\Omega' = \Omega - P_2 \omega'^{-1} P_2^T = \Omega - B_2 \omega'^{-1} B_2^T. \quad (9)$$

$\Omega'$  has three eigenvalues of order  $m^2/(M^2/\mu)$  and two of order  $M^2/\mu$ . Contributions from the term  $-B_2 \omega'^{-1} B_2^T$  in (9) are negligible as they are of order  $m^2/M$ . Thus  $\Omega' \approx \Omega$  and the light sector mass matrix has the form:

$$\begin{aligned} M_{Light} &= \Omega' \approx \Omega \\ &= \begin{pmatrix} 0_{3 \times 3} & \tilde{m}_{3 \times 2} \\ (\tilde{m}_{3 \times 2})^T & \tilde{M} \end{pmatrix}, \end{aligned} \quad (10)$$

in the basis  $(\nu_e, \nu_{\mu}, \nu_{\tau}, \nu'_{1R}^c, \nu'_{2R}^c)$ . In (10) the matrix  $\tilde{m}$  contains rotated entries of the Dirac mass matrix  $m_{LR}$  and  $\tilde{M} = \text{diag}(\lambda_1, \lambda_2)$  with the lighter eigenvalues of  $\omega_2^{diag}$  denoted as  $\lambda_{1,2} \sim M^2/\mu$ . Block diagonalising  $M_{Light}$  gives:

$$M_{Light} = \mathcal{R} \begin{pmatrix} \tilde{m}_{3 \times 2} \tilde{M}^{-1} (\tilde{m}_{3 \times 2})^T & 0 \\ 0 & \tilde{M} \end{pmatrix} \mathcal{R}^T, \quad (11)$$

with:

$$\mathcal{R} = \begin{pmatrix} I_{3 \times 3} & \tilde{P}_{3 \times 2} \\ -(\tilde{P}_{3 \times 2})^T & I_{2 \times 2} \end{pmatrix}, \quad (12)$$

and  $\tilde{P} = \tilde{m} \tilde{M}^{-1}$ . Eq. (11) demonstrates that  $\lambda_{1,2} \sim M^2/\mu$  are approximate eigenvalues of  $M_{\nu(9 \times 9)}$  with eigenvectors containing mostly  $\nu'_{1,2R}^c$  and a small  $\nu_{\alpha L}$ ,  $\alpha = e, \mu, \tau$ , component. The remaining three eigenvalues are found by diagonalising  $\tilde{m}_{3 \times 2} \tilde{M}^{-1} (\tilde{m}_{3 \times 2})^T$  and are thus  $\sim m^2/(M^2/\mu)$ . The eigenvectors are predominantly composed of the states  $\nu_{\alpha L}$ . Mixing between the states  $\nu_{\alpha L}$  and  $\nu'_{1,2R}^c$  is controlled by  $\tilde{P}$  and is of order  $m/(M^2/\mu)$ .

#### IV. SINGULAR DOUBLE SEE-SAW MECHANISM SCALES

The five lightest mass eigenstates generated by the singular double see-saw mechanism are structured in a suitable manner to realise a  $3 + 2$  neutrino spectrum. The scales required to make this  $3 + 2$  spectrum experimentally feasible are now discussed.

LSND requires mass-squared differences of  $\sim 1 - 10$  eV<sup>2</sup>. In the above model the relevant scales for LSND are set by  $\lambda_{1,2} \sim M^2/\mu$ . Consequently  $M^2/\mu \sim 1$  eV is required and the LSND result alone permits a freedom to scale both  $M$  and  $\mu$ . As  $M \sim \langle X_R \rangle$  is related to the mass of the gauge bosons coupled to the right-handed currents it is experimentally constrained. Considerations of the  $K_L - K_S$  mass difference have given the bound  $M_{W_R} \geq 1.6$  TeV [37], which implies  $M \gtrsim 1.6$  TeV, assuming that the coupling constants are not unreasonably small. This in turn gives  $\mu \gtrsim 10^{15}$  GeV. Note that if  $M \sim 1$  TeV then  $\mu \sim 10^{15}$  GeV which hints that the  $S_i$ 's may acquire mass at a GUT scale. The bounds on  $M$  and  $\mu$  decouple the eigenstates with masses of order  $M$  and  $\mu$  from the low energy phenomenology.

The predominantly active states have a mass of order  $m^2/(M^2/\mu)$ . The atmospheric neutrino bound of  $\sqrt{\Delta m_{atm}^2} \approx 5 \times 10^{-2}$  eV together with the relationship  $M^2/\mu \sim 1$  eV implies  $m \gtrsim 10^{-1}$  eV. With  $m \sim 10^{-1}$  eV and  $M^2/\mu \sim 1$  eV the active-sterile mixing from  $\tilde{P}$  is of order  $10^{-1}$ .

To fit the atmospheric and solar oscillation data in terms of active flavour oscillations requires:

$$\tilde{m}_{3 \times 2} \tilde{M}^{-1} (\tilde{m}_{3 \times 2})^T = U M^{diag} U^T, \quad (13)$$

where the  $3 \times 3$  mixing matrix  $U$  has the approximate form of the experimentally measured  $U_{PMNS}$  matrix. The full  $5 \times 5$  mixing matrix is:

$$U_{5 \times 5} = \begin{pmatrix} I_{3 \times 3} & \tilde{P}_{3 \times 2} \\ -(\tilde{P}_{3 \times 2})^T & I_{2 \times 2} \end{pmatrix} \begin{pmatrix} U & 0 \\ 0 & I_{2 \times 2} \end{pmatrix}. \quad (14)$$

Previous works suggest that taking  $\Delta m_{14}^2 \sim 1$  eV<sup>2</sup> and  $\Delta m_{15}^2 > 8$  eV<sup>2</sup> can enhance the LSND signal and alleviate the mixing matrix element bounds applicable to  $3 + 1$  models from the other short base-line experiments [21, 22]. To demonstrate the type of active-sterile mixing matrix elements obtained in this model we have numerically determined them for some mass values. Taking  $\theta_{12} = \pi/6$ ,  $\theta_{13} = 0$  and  $\theta_{23} = \pi/4$  for the PMNS angles we took the predominantly active mass eigenvalues to be:

$$\begin{aligned} M^{diag} \\ = U^T \tilde{m}_{3 \times 2} \tilde{M}^{-1} (\tilde{m}_{3 \times 2})^T U \\ = \text{diag}(0, 0.014, 0.045), \end{aligned} \quad (15)$$

where the masses are in eV. The diagonalisation of (15) is enforced with  $\Delta m_{41}^2 = 0.92$  eV<sup>2</sup> and  $\Delta m_{51}^2$  set at the best fit value obtained in [22],  $\Delta m_{51}^2 = 22$  eV<sup>2</sup>. The resulting five neutrino mixing matrix is:

$$U_{\alpha i} = \begin{pmatrix} 0.866 & 0.500 & 0 & -0.094 & 0.032 \\ -0.359 & 0.612 & 0.707 & -0.292 & 0.145 \\ 0.359 & -0.612 & 0.707 & -0.062 & 0.067 \\ 0 & 0.188 & 0.250 & 1 & 0 \\ 0 & -0.064 & -0.150 & 0 & 1 \end{pmatrix} \quad (16)$$

These values are to be compared with the best fit values  $U_{e4} = 0.121$ ,  $U_{e5} = 0.036$ ,  $U_{\mu 4} = 0.204$  and  $U_{\mu 5} = 0.224$  obtained in [22]. Both the electron and muon neutrinos couple more strongly to the fourth mass eigenstate in (16), though reducing  $M_5$  increases the elements  $|U_{e,\mu 5}|$ , as expected given the form of  $\tilde{P}$ . The greatest deviation from the values in [22] occurs for the elements  $U_{\mu 4,5}$ . When calculating the probability  $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$  relevant for LSND the mixing matrix elements always occur in the combinations  $U_{e4}U_{\mu 4}$  and  $U_{e5}U_{\mu 5}$ . The values in (16) give  $|U_{e4}U_{\mu 4}| = 2.7 \times 10^{-2}$  and  $|U_{e5}U_{\mu 5}| = 4.6 \times 10^{-3}$  whilst the values in [22] give  $2.4 \times 10^{-2}$  and  $8.1 \times 10^{-3}$  respectively. The smaller value  $|U_{e4}| = 0.094 < 0.121$  is seen to compensate somewhat for the deviation of  $|U_{\mu 4}| = 0.292 > 0.204$ . With the values in (16) the contribution of the heavier sterile state to the LSND signal is lower than that obtained with the best fit values in [22]. The contribution is still large enough however to enhance significantly the LSND signal relative to a  $3 + 1$  oscillation pattern. The above numbers are presented as an example only and the compatibility of this model with the best fit values in [22] depends on the size of the light mass eigenvalues. Deviations from the values  $\theta_{12} = \pi/6$ ,  $\theta_{13} = 0$  and  $\theta_{23} = \pi/4$  also shift the active-sterile mixing but the dependence of this mixing on the size of the light eigenvalues is generally stronger. We emphasise that within this framework, knowledge of the mass eigenvalues and the experimentally extractable elements of the  $3 \times 3$  PMNS matrix  $U$  specifies the size of the active-sterile mixing.

#### V. DISCUSSION

From a cosmological point of view this model, along with all models that attempt to explain LSND by the addition of sterile neutrinos, risks thermalising the sterile states and disrupting the standard BBN. We adopt the view that an adjustment of the standard paradigm will be required if the oscillation interpretation of the LSND result is confirmed, alleviating the need to comply with the derived bounds of  $N_\nu < 4$  from the  ${}^4\text{He}$  abundance [38, 39] and  $\sum m_\nu < 0.7 - 1.0$  eV from cosmological data [40].

We have not commented on the origin of the singularity in the singlet Majorana mass matrix. Many methods of obtaining singularities in mass matrices exist in the literature (for example [41, 42, 43]). Some methods which may be relevant for application to our model include a horizontal gauge symmetry in the singlet sector [44] and the supersymmetric realisation of a singular Majorana mass matrix of Du and Liu [45]. The method advocated in [46] to generate arbitrary mass matrix texture zeros could also be employed.

We have shown that the singular double see-saw mechanism can be employed to generate a  $3 + 2$  neutrino spectrum within a left-right symmetric framework. The resulting active-sterile mixing is found to be determined once the mass eigenvalues and the submatrix of the  $5 \times 5$  mixing matrix corresponding to the PMNS matrix are specified. From a model building point of view this result may be of interest if MiniBooNe confirms the oscillation interpretation of the LSND re-

sult.

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